Scalar $f_0(980)$ and $\sigma(500)$ meson exchange in ϕ decays into $\pi^0\pi^0\gamma$

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Abstract. The complementarity between chiral perturbation theory and the linear sigma model is exploited to study $\pi^0\pi^0$ production in ϕ radiative decays, where the effects of the $f_0(980)$ scalar resonance, and those of its more controversial $\sigma(500)$ partner, should become manifest via the $\phi \to K^+K^-(\gamma) \to \pi^0\pi^0\gamma$ decay chain. The recently reported data on $\phi \to \pi^0\pi^0\gamma$ coming from the VEPP-2M e^+e^- collider in Novosibirsk and the DAΦNE ϕ-factory in Frascati can be reasonably described in our approach, which we propose as a promising first step towards more detailed analyses. The $f_0(980)$ contribution, which appears as a moderately narrow peak at the high part of the dipion mass spectrum, can be interpreted as the isoscalar member of the scalar nonet with a large $f_0 K\bar{K}$ coupling and an $f_0\pi\pi$ coupling suppressed by almost ideal σ–f_0 mixing. Indeed, the mixing angle in the flavor basis is found to be $\phi_S \approx -6°$, if the f_0 propagator is approximated by a simple Breit–Wigner expression, or $\phi_S \approx -9°$, if an improved two-channel analysis is performed. The $\sigma(500)$ resonance, which is then strongly coupled to pion pairs, yields a tiny contribution because, in our approach, its coupling to kaon pairs is proportional to $m_\sigma^2 - m_K^2$ and thus quite small.

1 Introduction

The radiative decays of low mass vector mesons into two neutral pseudoscalars, $V \to P^0 P^0 \gamma$, are known to be a useful tool to investigate the complicated dynamics governing meson physics around 1 GeV. Particularly interesting are the decays proceeding mainly by the exchange of scalar resonances because of the unconventional [1] and controversial nature of these states, as exemplified, for instance, in the very recent analysis by Close and Törnqvist [2]. Experimental data on the $\phi \to \pi^0\pi^0\gamma$ decays where, after the emission of a photon, a K^+K^- pair rescatters into $\pi^0\pi^0$ in a process dominated by the exchange of isoscalar scalar resonances in the s-channel have thus been watched with great interest [3,4].

The first measurements of this $\phi \to \pi^0\pi^0\gamma$ decay have been reported by the SND and CMD-2 Collaborations. For the branching ratio, they obtain

$$B(\phi \to \pi^0\pi^0\gamma)$$
$$= \begin{cases} (1.22 \pm 0.10 \pm 0.06) \times 10^{-4} \text{ SND [5]}, \\ (0.92 \pm 0.08 \pm 0.06) \times 10^{-4} \text{ CMD-2 [6]}, \end{cases} \quad (1)$$

for $m_{\pi^0\pi^0} > 700$ MeV in the latter case. More recently, the KLOE Collaboration has measured [7]

$$B(\phi \to \pi^0\pi^0\gamma) = (1.09 \pm 0.03 \pm 0.05) \times 10^{-4}, \quad (2)$$

in agreement with (1) but with a considerably smaller error. In all cases, the spectrum is clearly peaked at $m_{\pi^0\pi^0} \simeq 970$ MeV, as expected from an important $f_0(980)$ contribution. This and other radiative vector meson decays are now being further investigated at the Frascati ϕ-factory DAΦNE [8] with higher accuracy (see for example [9]).

On the theoretical side, the $\phi \to \pi^0\pi^0\gamma$ decay has been considered by a number of authors [10–17]. Early calculations of the vector meson dominance (VMD) amplitude for these processes, i.e. the contributions proceeding through the decay chain $\phi \to \pi^0\rho^0 \to \pi^0\pi^0\gamma$, were summarized in [10]. The ϕ decay vertex is forbidden by the Zweig rule and the width and branching ratios predicted by VMD, $\Gamma(\phi \to \pi^0\pi^0\gamma)_{\rm VMD} = 51$ eV and $B(\phi \to \pi^0\pi^0\gamma)_{\rm VMD} = 0.12 \times 10^{-4}$ [10], are found to be substantially smaller than the experimental results quoted in (1) and (2). Moreover the VMD spectrum peaks at low $m_{\pi\pi}$ values well below the $f_0(980)$ resonance mass.

The possibility of an enhancement in this branching ratio through the $\phi \to K^+K^-(\gamma) \to \pi^0\pi^0\gamma$ mechanism [3,4] was pointed out in [10] and further discussed in [11] in a chiral perturbation theory (ChPT) context enlarged to include on-shell vector mesons. This formalism gives well-defined predictions for the various $V \to P^0 P^0 \gamma$ de-

cays in terms of $P^+P^- \to P^0P^0$ rescattering amplitudes, which are easily calculated in strict ChPT, and a loop integral over the intermediate P^+P^- pair. In this approach, the $\phi \to \pi^0\pi^0\gamma$ decay is dominated by (Zweig rule allowed) kaon loops leading to $\Gamma(\phi \to \pi^0\pi^0\gamma)_\chi = 224\,\text{eV}$, while (Zweig rule forbidden) pion loop contributions are strongly suppressed. The corresponding spectrum peaks at moderate values of $m_{\pi\pi}$ but the interference between this kaon loop contribution and the previous VMD amplitude turns out to be quite small, leading globally to $\Gamma(\phi \to \pi^0\pi^0\gamma)_{\text{VMD}+\chi} = 269\,\text{eV}$ and $B(\phi \to \pi^0\pi^0\gamma)_{\text{VMD}+\chi} = 0.61 \times 10^{-4}$ [11], which are still below the experimental results quoted in (1) and (2). Additional contributions are thus certainly required. The most natural candidates for closing this gap between theory and experiment are the contributions coming from the exchange of scalar resonances, such as the well-established $f_0(980)$ and the more controversial $\sigma(500)$ (or $f_0(400{-}1200)$) mesons [18].

The purpose of this paper is to study the effects of the low mass scalar states in the $\phi \to \pi^0\pi^0\gamma$ decays following the ChPT inspired context introduced in [19–21] to account similarly for the scalar exchange contributions to $\phi \to \pi^0\eta\gamma$ and $\rho, \omega \to \pi^0\pi^0\gamma$. In this context, one takes advantage of the common origin of ChPT and the LσM to improve the chiral loop predictions for $V \to P^0P^0\gamma$ exploiting the complementarity of both approaches for these specific processes. As a result, a simple analytic amplitude, $\mathcal{A}(\phi \to \pi^0\pi^0\gamma)_{\text{L}\sigma\text{M}}$, will be obtained which includes the effects of the scalar meson poles and also shows the appropriate behavior expected from ChPT at low dipion invariant masses. There also exists the contribution to $\phi \to \pi^0\pi^0\gamma$ coming from the previously mentioned vector meson exchange. This VMD amplitude, $\mathcal{A}(\phi \to \pi^0\pi^0\gamma)_{\text{VMD}}$, is well known and scarcely interesting but has to be added to $\mathcal{A}(\phi \to \pi^0\pi^0\gamma)_{\text{L}\sigma\text{M}}$, i.e. to the relevant amplitudes containing the scalar meson effects, in order to compare with available and forthcoming data. We will conclude that data on the $\phi \to \pi^0\pi^0\gamma$ channel can contribute decisively to the improvement of our knowledge of the $f_0(980)$ scalar resonance, as well as on some specific features of its elusive and controversial partner $\sigma(500)$.

2 Chiral loop contributions to $\phi \to \pi^0\pi^0\gamma$

The vector meson initiated $V \to P^0P^0\gamma$ decays cannot be treated in strict chiral perturbation theory (ChPT). This theory has to be extended to incorporate on-shell vector meson fields. At lowest order, this may easily be achieved by means of the $\mathcal{O}(p^2)$ ChPT Lagrangian

$$\mathcal{L}_2 = \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U + M(U + U^\dagger)\rangle, \qquad (3)$$

where $U = \exp(i(2^{1/2})P/f)$, P is the usual pseudoscalar octet matrix and, at this order, $M = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$ and $f = f_\pi = 92.4\,\text{MeV}$. The covariant derivative, now enlarged to include vector mesons, is defined as

$$D_\mu U = \partial_\mu U - ieA_\mu[Q, U] - ig[V_\mu, U]$$

with $Q = \text{diag}(2/3, -1/3, -1/3)$ being the quark charge matrix and V_μ the additional matrix containing the nonet of vector meson fields. We follow the conventional normalization for the vector nonet matrix such that the diagonal elements are $(\rho^0 + \omega_0)/(2^{1/2})$, $(-\rho^0 + \omega_0)/(2^{1/2})$ and ϕ_0, where $\omega_0 = (u\bar{u} + d\bar{d})/(2^{1/2})$ and $\phi_0 = s\bar{s}$ stand for the ideally mixed states. The physical ϕ-field is approximately $\phi_0 + \epsilon\omega_0$, where $\epsilon \simeq +0.059 \pm 0.004$ accounts for the ω–ϕ mixing angle in the flavor basis [18].

One easily observes that there is no tree-level contribution from the Lagrangian (3) to the $\phi \to \pi^0\pi^0\gamma$ amplitude and that, at the one-loop level, one needs to compute the set of diagrams shown in [11]. We do not take into account pion loop contributions here since they proceed only through the small ϵ piece of the ϕ-field (because of the Zweig rule) which is further suppressed by G-parity, as in $\omega \to \pi^+\pi^-$. These pion loop contributions are known to be negligible as compared to those from kaon loops proceeding through the Zweig rule allowed piece, $\phi_0 = s\bar{s}$, not restricted by G-parity arguments [11].

A straightforward calculation leads to the following finite amplitude for $\phi(q^*, \epsilon^*) \to \pi^0(p)\pi^0(p')\gamma(q, \epsilon)$ (see [11] for further details):

$$\mathcal{A}(\phi \to \pi^0\pi^0\gamma)_\chi = \frac{eg_s}{2\pi^2 m_{K^+}^2}\{a\}L(m_{\pi^0\pi^0}^2)$$
$$\times \mathcal{A}(K^+K^- \to \pi^0\pi^0)_\chi, \qquad (4)$$

where $\{a\} = (\epsilon^* \cdot \epsilon)(q^* \cdot q) - (\epsilon^* \cdot q)(\epsilon \cdot q^*)$, $m_{\pi^0\pi^0}^2 \equiv s \equiv (p + p')^2 = (q^* - q)^2$ is the invariant mass of the final dipion system and $L(m_{\pi^0\pi^0}^2)$ is the loop integral function defined by [3, 4, 11, 15, 22]

$$L(m_{\pi^0\pi^0}^2) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right]$$
$$+ \frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right)\right], \qquad (5)$$

with

$$f(z) = \begin{cases} -\left[\arcsin\left(\dfrac{1}{2\sqrt{z}}\right)\right]^2, & z > \dfrac{1}{4}, \\[2ex] \dfrac{1}{4}\left(\log\dfrac{\eta_+}{\eta_-} - i\pi\right)^2, & z < \dfrac{1}{4}, \end{cases}$$

$$g(z) = \begin{cases} \sqrt{4z - 1}\,\arcsin\left(\dfrac{1}{2\sqrt{z}}\right), & z > \dfrac{1}{4}, \\[2ex] \dfrac{1}{2}\sqrt{1 - 4z}\left(\log\dfrac{\eta_+}{\eta_-} - i\pi\right), & z < \dfrac{1}{4}, \end{cases} \qquad (6)$$

and $\eta_\pm = (1/2)(1 \pm (1 - 4z)^{1/2})$, $a = m_\phi^2/m_{K^+}^2$ and $b = m_{\pi^0\pi^0}^2/m_{K^+}^2$. The coupling constant g_g comes from the strong decay amplitude $\mathcal{A}(\phi \to K^+K^-) = g_g\epsilon^* \cdot (p_+ - p_-)$ and takes the value $|g_s| \simeq 4.6$ to agree with $\Gamma(\phi \to K^+K^-)_{\text{exp}} = 2.19\,\text{MeV}$ [18]. This coupling is the part beyond standard ChPT which we have fixed phenomenologically.

The four-pseudoscalar amplitude is instead a standard ChPT amplitude which is found to depend linearly on the variable $s = m_{\pi^0\pi^0}^2$ only:

$$\mathcal{A}(K^+K^- \to \pi^0\pi^0)_\chi = \frac{s}{4f_\pi f_K}, \qquad (7)$$

with $f_K = 1.22 f_\pi$. It is important to notice that in deriving this ChPT amplitude no use has been made of the relation $s + t + u = 2m_K^2 + 2m_\pi^2$ valid only for on-shell particles but not in our case with off-shell kaons in the chiral loop. Thanks to this genuine t and u independence, the chiral amplitude (7) factorizes in (4).

Integrating the invariant mass distribution for the $\phi \to \pi^0\pi^0\gamma$ decay over the whole physical region one obtains $\Gamma(\phi \to \pi^0\pi^0\gamma)_\chi = 219\,\mathrm{eV}$ and

$$B(\phi \to \pi^0\pi^0\gamma)_\chi = 0.49 \times 10^{-4}. \qquad (8)$$

These results improve the prediction for this process given in [11] where $SU(3)$-breaking effects were ignored[1].

3 Scalar meson exchange in $\phi \to \pi^0\pi^0\gamma$

We now turn to the contributions coming from scalar resonance exchange. From a ChPT perspective their effects are encoded in the low energy constants of the higher order pieces of the ChPT Lagrangian. But the effects of the $f_0(980)$ – and its lower mass partner $\sigma(500)$ meson, if confirmed – should become manifest in the $\phi \to \pi^0\pi^0\gamma$ decays not as a constant term but rather through more complex resonant amplitudes. In this section we propose an amplitude which not only obeys the ChPT dictates in the lowest part of the $\pi^0\pi^0$ spectrum, as it must, but also generates the scalar meson effects for the higher part of the spectrum where the resonant poles should dominate.

The linear sigma model (LσM) [23–25], which we take as a first guidance probably requiring future refinements, will be shown to be particularly appropriate for our purposes. It is a well-defined $U(3) \times U(3)$ chiral model which incorporates ab initio both the nonet of pseudoscalar mesons together with its chiral partner, the scalar mesons nonet. In this context, the $V \to P^0P^0\gamma$ decays proceed through a loop of charged pseudoscalar mesons emitted by the initial vector. Because of the additional emission of a photon, charged pseudoscalar pairs with the initial $J^{PC} = 1^{--}$ quantum numbers can rescatter into $J^{PC} = 0^{++}$ pairs of charged or neutral pseudoscalars. For the $\phi \to \pi^0\pi^0\gamma$ decay the contributions from charged pion loops are again negligible compared to those from kaon loops and do not need to be considered for the same reasons as discussed in the previous section. The elusive $\sigma(500)$ and the clearly established $f_0(980)$ scalar resonances are then expected, at least in principle, to play the central role in this $K^+K^- \to \pi^0\pi^0$ rescattering process. Their contributions will be conveniently parameterized in terms of LσM amplitudes compatible with ChPT for low dipion invariant masses.

The $K^+K^- \to \pi^0\pi^0$ amplitude in the LσM turns out to be

$$\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\mathrm{L}\sigma\mathrm{M}} = g_{K^+K^-\pi^0\pi^0}$$
$$- \frac{g_{\sigma K^+K^-} g_{\sigma\pi^0\pi^0}}{s - m_\sigma^2} - \frac{g_{f_0 K^+K^-} g_{f_0\pi^0\pi^0}}{s - m_{f_0}^2}$$
$$- \frac{g_{\kappa^\mp K^\pm\pi^0}^2}{t - m_\kappa^2} - \frac{g_{\kappa^\mp K^\pm\pi^0}^2}{u - m_\kappa^2}, \qquad (9)$$

where the various coupling constants are fixed within the model and can be expressed in terms of f_π, f_K, the masses of the pseudoscalar and scalar mesons involved in the process, and the scalar meson mixing angle in the flavor basis ϕ_S [26,27]. In particular, the $g_{K^+K^-\pi^0\pi^0}$ coupling accounting for the constant four-pseudoscalar amplitude can be expressed in a more convenient form by imposing that the amplitude $\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\mathrm{L}\sigma\mathrm{M}}$ vanishes in the soft-pion limit (either $p \to 0$ or $p' \to 0$). Then, the amplitude (9) can be rewritten as the sum of three terms, each one depending only on s, t and u:

$$\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\mathrm{L}\sigma\mathrm{M}}$$
$$\equiv \mathcal{A}_{\mathrm{L}\sigma\mathrm{M}}^s(s) + \mathcal{A}_{\mathrm{L}\sigma\mathrm{M}}^t(t) + \mathcal{A}_{\mathrm{L}\sigma\mathrm{M}}^u(u)$$
$$= \frac{s - m_\pi^2}{2f_\pi f_K}\left[\frac{m_K^2 - m_\sigma^2}{D_\sigma(s)}c\phi_S(c\phi_S\right.$$
$$\left. - \sqrt{2}s\phi_S) + \frac{m_K^2 - m_{f_0}^2}{D_{f_0}(s)}s\phi_S(s\phi_S + \sqrt{2}c\phi_S)\right]$$
$$+ \frac{t - m_K^2}{4f_\pi f_K}\frac{m_\pi^2 - m_\kappa^2}{D_\kappa(t)} + \frac{u - m_K^2}{4f_\pi f_K}\frac{m_\pi^2 - m_\kappa^2}{D_\kappa(u)}, \qquad (10)$$

where $D_S(s) = s - m_S^2 + im_S\Gamma_S$ are the $S = \sigma, f_0$ propagators – similar expressions hold for $D_\kappa(t, u)$ – and $(c\phi_S, s\phi_S) \equiv (\cos\phi_S, \sin\phi_S)$. Notice that we start using simple Breit–Wigner expressions for the propagators in spite of the well-known difficulties associated to threshold effects for the f_0. Moreover, the amplitude (10) is not strictly unitary. However, we will not discuss this issue in detail but refer the reader to [28] where a careful study of full unitarity in $K\pi$ scattering in the framework of the LσM can be found. We should stress that the main concern of our analysis is to explicitly incorporate the scalar resonances and that, for this specific purpose, the tree-level (and not fully unitary) amplitude (10) with simple expressions for the scalar propagators will be shown to be a reasonable starting point. A more sophisticated treatment of the f_0 propagator [4,29,30] accounting for the $K\bar{K}$ threshold effects $(m_{f_0} \simeq 2m_K)$ will be discussed later on.

A few remarks on the four-pseudoscalar amplitudes in (9) and (10) and on their comparison with the ChPT amplitude in (7) are of interest.

(i) For $m_S \to \infty$ $(S = \sigma, f_0, \kappa)$, the L$\sigma$M amplitude (10) reduces to

$$\frac{s - m_\pi^2}{2f_\pi f_K} + \frac{t + u - 2m_K^2}{4f_\pi f_K},$$

which would coincide with the ChPT amplitude (7) if the on-shell condition $s + t + u = 2m_\pi^2 + 2m_K^2$ could be invoked.

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[1] In the good $SU(3)$ limit, i.e. with the chiral amplitude $\mathcal{A}(K^+K^- \to \pi^0\pi^0)_\chi = s/4f_\pi f_K \to s/4f_\pi^2$ and $g_s \to g \simeq 4.2$ for the $\rho\pi\pi$ strong coupling constant used in [11], one obtains $B(\phi \to \pi^0\pi^0\gamma)_\chi = 0.54 \times 10^{-4}$

As shown in (9), $\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\mathrm{L\sigma M}}$ consists of a constant four-pseudoscalar vertex plus three terms whose s, t, u dependence is generated by the scalar propagators $D_S(s, t, u)$. Combining each one of these three terms with its corresponding part from the first, the four-pseudoscalar vertex leads immediately to (10). For $m_S \to \infty$, this amplitude is linear in the s, t, u variables, mimics perfectly the effects of the derivative and massive terms in the Lagrangian (3) and leads to the ChPT amplitude (7) once $s + t + u = 2m_K^2 + 2m_\pi^2$ is used. This corresponds to the aforementioned complementarity between ChPT and the LσM, thus making the whole analysis quite reliable.

(ii) However, the relation $s + t + u = 2m_K^2 + 2m_\pi^2$ cannot be invoked when plugging our LσM amplitude (10) into the loop of *virtual* kaons leading to the $\phi \to \pi^0\pi^0\gamma$ decay amplitude. This is in sharp contrast with the processes studied in [19–21], governed by t, u-independent LσM amplitudes, and thus requires a new treatment for our case. To this aim, one has to distinguish between the $\mathcal{A}_{\mathrm{L\sigma M}}^s$ term in (10) and the $\mathcal{A}_{\mathrm{L\sigma M}}^t + \mathcal{A}_{\mathrm{L\sigma M}}^u$ terms of the same amplitude. The former, being only s-dependent, can be directly introduced in (4) instead of the $\mathcal{A}(K^+K^- \to \pi^0\pi^0)_\chi$ amplitude. Fortunately, this term contains most of the relevant dynamics due to the scalar resonances – the $f_0(980)$ and, eventually, the $\sigma(500)$ poles – for the process $\phi \to \pi^0\pi^0\gamma$ where the dipion mass spectrum covers the range $4m_\pi^2 \leq m_{\pi\pi}^2 \leq m_\phi^2$. The remaining two terms $\mathcal{A}_{\mathrm{L\sigma M}}^t + \mathcal{A}_{\mathrm{L\sigma M}}^u$, being t, u-dependent, cannot be simply plugged and evaluated into the loop integral[2] but their contribution to the $\phi \to \pi^0\pi^0\gamma$ amplitude can be identified and reasonably estimated. Indeed, the κ contribution to the $\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\mathrm{L\sigma M}}$ amplitude can be fixed by subtracting from the chiral-loop amplitude in (7) the contributions from the σ and f_0 terms in (10) taking $m_{\sigma, f_0} \to \infty$. The resulting expression corresponds to the desired κ contribution in the $m_\kappa \to \infty$ limit. The difference between this κ contribution for $m_\kappa \to \infty$ and the same contribution for a κ of finite mass can be considered as negligible due to the high mass of the κ and the lack of scalar poles in the t- and u-channel of the $\phi \to \pi^0\pi^0\gamma$ decay process[3]. In other words, while the $K^+K^- \to \pi^0\pi^0$ rescattering amplitude evaluated at lowest order in ChPT is necessarily a poor approximation to the (pole dominated) s-channel dynamics in $\phi \to \pi^0\pi^0\gamma$ decays, the same lowest order amplitude can be taken as a reasonable estimate for the t- and u-channel contributions where the κ pole position cannot be kinematically approached. Therefore, an improved expression for the chiral-loop amplitude in (7) is the following:

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[2] A more involved four-propagator loop integral would be required to compute the contributions of the t, u-channel in a precise way

[3] The κ has been identified in the literature as a broad resonance with a mass around $900\,\mathrm{MeV}$ (see for example [26]) or with the $K_0^*(1430)$ scalar resonance as in [27]. A model-independent analysis based on the analytic continuation of πK scattering finds that there is room for a $K_0^*(1430)$ scalar state but not for a $\kappa(900)$ one [31]

$$\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\mathrm{L\sigma M}} = \frac{m_\pi^2 - s/2}{2f_\pi f_K}$$
$$+ \frac{s - m_\pi^2}{2f_\pi f_K}\left[\frac{m_K^2 - m_\sigma^2}{D_\sigma(s)}\mathrm{c}\phi_S(\mathrm{c}\phi_S - \sqrt{2}\mathrm{s}\phi_S)\right.$$
$$\left.+ \frac{m_K^2 - m_{f_0}^2}{D_{f_0}(s)}\mathrm{s}\phi_S(\mathrm{s}\phi_S + \sqrt{2}\mathrm{c}\phi_S)\right]. \quad (11)$$

As desired, it has no t, u dependence and can be thus plugged in (4) instead of $\mathcal{A}(K^+K^- \to \pi^0\pi^0)_\chi$. One then obtains the s-dependent amplitude

$$\mathcal{A}(\phi \to \pi^0\pi^0\gamma)_{\mathrm{L\sigma M}} = \frac{eg_s}{2\pi^2 m_{K^+}^2}\{a\}L(s)$$
$$\times \mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\mathrm{L\sigma M}}, \quad (12)$$

which will be used from now on. Notice that for large scalar masses one recovers (4), that the $f_0(980)$ (and, eventually, $\sigma(500)$) s-channel poles do now appear, and that the remaining term, $(m_\pi^2 - s/2)/2f_\pi f_K$, accounts for κ exchange effects in the t, u-channel where, as stated before, no poles are approached and has thus been approximated by the corresponding part of the simple chiral-loop amplitude.

(iii) The large widths of the scalar resonances break chiral symmetry if they are naively introduced in (9), an effect already noticed in [32]. Accordingly, we introduce the $\sigma(500)$ and $f_0(980)$ widths in the propagators only *after* chiral cancellation of constant terms in the amplitude. In this way the pseudo-Goldstone nature of pions is preserved.

(iv) The $\pi^0\pi^0$ invariant mass spectrum for the $\phi \to \pi^0\pi^0\gamma$ decay covers the region where the presence of the $f_0(980)$ (and the $\sigma(500)$) meson(s) should become manifest. Because of the presence of the corresponding propagators in (11) – closely linked to the ChPT amplitude and thus expected to account for the lowest part of the $\pi^0\pi^0$ spectrum – one should also be able to reproduce the effects of the f_0 (and the $\sigma(500)$) pole(s) at higher $\pi^0\pi^0$ invariant mass values.

4 Vector meson exchange in $\phi \to \pi^0\pi^0\gamma$

In addition to the LσM contributions, which can be viewed as an improved version of the chiral-loop predictions, the analysis should be extended to include vector meson exchange in the t- and u-channel. These VMD contributions were already considered in [10]. In this framework, $\phi \to \pi^0\pi^0\gamma$ proceeds through ω–ϕ mixing followed by the exchange of intermediate ρ^0 mesons in the direct or crossed channel of the $\phi \to \pi^0\rho^0 \to \pi^0\pi^0\gamma$ decay chain.

In order to describe these vector meson contributions we use the $SU(3)$-symmetric Lagrangians

$$\mathcal{L}_{\mathrm{VVP}} = \frac{G}{\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\langle\partial_\mu V_\nu \partial_\alpha V_\beta P\rangle,$$
$$\mathcal{L}_{\mathrm{V\gamma}} = -4f^2 egA_\mu\langle QV^\mu\rangle, \quad (13)$$

where $G = (3g^2)/(4\pi^2 f)$ is the $\omega\rho\pi$ coupling constant and $|g| \simeq 4.0$ as follows from various ρ and ω decay data [12,18]. The VMD amplitude for $\phi(q^*, \epsilon^*) \to \pi^0(p)\pi^0(p') \gamma(q, \epsilon)$ is then found to be

$$\mathcal{A}(\phi \to \pi^0\pi^0\gamma)_{\text{VMD}} \qquad (14)$$
$$= \frac{\epsilon}{3}\frac{G^2 e}{\sqrt{2}g}\left(\frac{P^2\{a\} + \{b(P)\}}{M_\rho^2 - P^2 - iM_\rho\Gamma_\rho} + \frac{P'^2\{a\} + \{b(P')\}}{M_\rho^2 - P'^2 - iM_\rho\Gamma_\rho}\right),$$

with $\{a\}$ as in (4) and a new amplitude

$$\{b(P)\} = -(\epsilon^* \cdot \epsilon)(q^* \cdot P)(q \cdot P) - (\epsilon^* \cdot P)(\epsilon \cdot P)(q^* \cdot q)$$
$$+ (\epsilon^* \cdot q)(\epsilon \cdot P)(q^* \cdot P)$$
$$+ (\epsilon \cdot q^*)(\epsilon^* \cdot P)(q \cdot P), \qquad (15)$$

where $P = p + q$ and $P' = p' + q$ are the momenta of the intermediate ρ^0 meson in the t- and u-channel, respectively. From this VMD amplitude one easily obtains $\Gamma(\phi \to \pi^0\pi^0\gamma)_{\text{VMD}} = 37 \text{ eV}$ and

$$B(\phi \to \pi^0\pi^0\gamma)_{\text{VMD}} = 8.3 \times 10^{-6}, \qquad (16)$$

in agreement with the results in [10,16] once the same numerical inputs are used. Further details on these contributions are given in the following section.

5 Final results

Our final results for $\mathcal{A}(\phi \to \pi^0\pi^0\gamma)$ are thus the sum of the VMD contribution in (14) plus the LσM contribution containing the scalar resonance effects in (12). The final $\pi^0\pi^0$ invariant mass distribution for the process is[4]

$$\frac{d\Gamma(\phi \to \pi^0\pi^0\gamma)}{dm_{\pi^0\pi^0}} = \frac{d\Gamma_{\text{L}\sigma\text{M}}}{dm_{\pi^0\pi^0}} + \frac{d\Gamma_{\text{VMD}}}{dm_{\pi^0\pi^0}} + \frac{d\Gamma_{\text{int}}}{dm_{\pi^0\pi^0}}, \quad (17)$$

where the LσM term (the scalar or *signal* contribution) is

$$\frac{d\Gamma_{\text{L}\sigma\text{M}}}{dm_{\pi^0\pi^0}}$$
$$= \frac{1}{2}\frac{\alpha}{192\pi^5}\frac{g_s^2}{4\pi}\frac{m_\phi^4}{m_{K^+}^4}\frac{m_{\pi^0\pi^0}}{m_\phi}\left(1 - \frac{m_{\pi^0\pi^0}^2}{m_\phi^2}\right)^3\sqrt{1 - \frac{4m_{\pi^0}^2}{m_{\pi^0\pi^0}^2}}$$
$$\times |L(m_{\pi^0\pi^0}^2)|^2|\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\text{L}\sigma\text{M}}|^2, \qquad (18)$$

with the four-pseudoscalar amplitude taken from (11). The VMD term (the vector or *background* contribution) and the contribution resulting from the interference of both amplitudes are given by

$$\frac{d\Gamma_{\text{[VMD,int]}}}{dm_{\pi^0\pi^0}} = \frac{1}{2}\frac{1}{256\pi^3}\frac{m_{\pi^0\pi^0}}{m_\phi}\left(1 - \frac{m_{\pi^0\pi^0}^2}{m_\phi^2}\right)\sqrt{1 - \frac{4m_{\pi^0}^2}{m_{\pi^0\pi^0}^2}}$$
$$\times \int_{-1}^{1}dx A_{\text{[VMD,int]}}(m_{\pi^0\pi^0}, x), \qquad (19)$$

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[4] In terms of the photon energy, $E_\gamma = (m_\phi^2 - m_{\pi^0\pi^0}^2)/(2m_\phi)$, the photonic spectrum is written as $d\Gamma/dE_\gamma = (m_\phi/m_{\pi^0\pi^0}) \times d\Gamma/dm_{\pi^0\pi^0}$

where one explicitly has

$$A_{\text{VMD}}(m_{\pi^0\pi^0}, x) \equiv \frac{1}{3}\sum_{\text{pol}}|\mathcal{A}_{\text{VMD}}|^2 = \frac{1}{3}\left(\frac{\epsilon}{3}\frac{G^2 e}{\sqrt{2}g}\right)^2$$

$$\times \left[\frac{1}{8}\left(m_{\pi^0}^8 - 2m_\phi^2 m_{\pi^0}^6 + 4\tilde{m}_\rho^4 m_{\pi^0}^4 + m_\phi^4 m_{\pi^0}^4\right.\right.$$
$$- 2\tilde{m}_\rho^2\tilde{m}_\rho^{*2}m_{\pi^0}^4 - 4\tilde{m}_\rho^6 m_{\pi^0}^2 - 2\tilde{m}_\rho^2 m_\phi^2 m_{\pi^0}^2 + 4\tilde{m}_\rho^4 m_\phi^2 m_{\pi^0}^2$$
$$- 4\tilde{m}_\rho^4\tilde{m}_\rho^{*2}m_{\pi^0}^2 + 2\tilde{m}_\rho^2 m_\phi^2\tilde{m}_\rho^{*2}m_{\pi^0}^2 + \tilde{m}_\rho^8 + \tilde{m}_\rho^4 m_\phi^4$$
$$+ 2\tilde{m}_\rho^4\tilde{m}_\rho^{*4} - 2\tilde{m}_\rho^6 m_\phi^2 + 2\tilde{m}_\rho^6\tilde{m}_\rho^{*2} - 2\tilde{m}_\rho^4 m_\phi^2\tilde{m}_\rho^{*2}\right)$$
$$\times \frac{1}{|D_\rho(\tilde{m}_\rho^2)|^2} + \frac{1}{16}\left(m_{\pi^0}^8 - \tilde{m}_\rho^2 m_\phi^2 m_{\pi^0}^4 + 2\tilde{m}_\rho^2\tilde{m}_\rho^{*2}m_{\pi^0}^4\right.$$
$$- m_\phi^2\tilde{m}_\rho^{*2}m_{\pi^0}^4 - 4\tilde{m}_\rho^2\tilde{m}_\rho^{*4}m_{\pi^0}^2 - 4\tilde{m}_\rho^4\tilde{m}_\rho^{*2}m_{\pi^0}^2$$
$$+ 4\tilde{m}_\rho^2 m_\phi^2\tilde{m}_\rho^{*2}m_{\pi^0}^2 + \tilde{m}_\rho^2\tilde{m}_\rho^{*6}$$
$$\left.+ 3\tilde{m}_\rho^2\tilde{m}_\rho^{*4} - \tilde{m}_\rho^2 m_\phi^2\tilde{m}_\rho^{*4} + \tilde{m}_\rho^6\tilde{m}_\rho^{*2} - \tilde{m}_\rho^4 m_\phi^2\tilde{m}_\rho^{*2}\right)$$
$$\left.\times 2\text{Re}\left(\frac{1}{D_\rho(\tilde{m}_\rho^2)D_\rho^*(\tilde{m}_\rho^{*2})}\right) + \left(\tilde{m}_\rho^2 \leftrightarrow \tilde{m}_\rho^{*2}\right)\right], \qquad (20)$$

and

$$A_{\text{int}}(m_{\pi^0\pi^0}, x) \equiv \frac{2}{3}\text{Re}\sum_{\text{pol}}\mathcal{A}_{\text{L}\sigma\text{M}}\mathcal{A}_{\text{VMD}}^*$$

$$= \frac{1}{3}\left(\frac{eg_s}{2\pi^2 m_{K^+}^2}\right)\left(\frac{\epsilon}{3}\frac{G^2 e}{\sqrt{2}g}\right)$$

$$\times 2\text{Re}\left\{L(m_{\pi^0\pi^0}^2)\mathcal{A}(K^+K^- \to \pi^0\pi^0)_{\text{L}\sigma\text{M}}\right.$$

$$\times \frac{1}{4}\left[\frac{\tilde{m}_\rho^2(\tilde{m}_\rho^2 + \tilde{m}_\rho^{*2} - 2m_{\pi^0}^2)^2 - m_\phi^2(\tilde{m}_\rho^2 - m_{\pi^0}^2)^2}{D_\rho^*(\tilde{m}_\rho^2)}\right.$$

$$\left.\left.+ \left(\tilde{m}_\rho^2 \leftrightarrow \tilde{m}_\rho^{*2}\right)\right]\right\}, \qquad (21)$$

with

$$P^2 \equiv \tilde{m}_\rho^2 = m_{\pi^0}^2 + \frac{m_\phi^2 - m_{\pi^0\pi^0}^2}{2}\left(1 - x\sqrt{1 - \frac{4m_{\pi^0}^2}{m_{\pi^0\pi^0}^2}}\right),$$

$$P'^2 \equiv \tilde{m}_\rho^{*2} = m_{\pi^0}^2 + \frac{m_\phi^2 - m_{\pi^0\pi^0}^2}{2}\left(1 + x\sqrt{1 - \frac{4m_{\pi^0}^2}{m_{\pi^0\pi^0}^2}}\right)$$

$$= 2m_{\pi^0}^2 + m_\phi^2 - m_{\pi^0\pi^0}^2 - \tilde{m}_\rho^2. \qquad (22)$$

The separate contributions from LσM, VMD and their interference, as well as the total result are shown in Fig. 1. In this figure, whose main purpose is to illustrate the possibilities of our approach, we use simple Breit–Wigner formulae for the σ and the f_0 propagators. We take $m_\sigma = 478 \text{ MeV}$ and $\Gamma_\sigma = 324 \text{ MeV}$, which are the central values of the measurements quoted in [33], $m_{f_0} = 980 \text{ MeV}$ and $\Gamma_{f_0} = 70 \text{ MeV}$, from [18], and $\phi_S = -6°$ for the scalar mixing angle. The global agreement with the data is rather good although the model will require further improvements. As expected, $f_0(980)$ scalar meson exchange
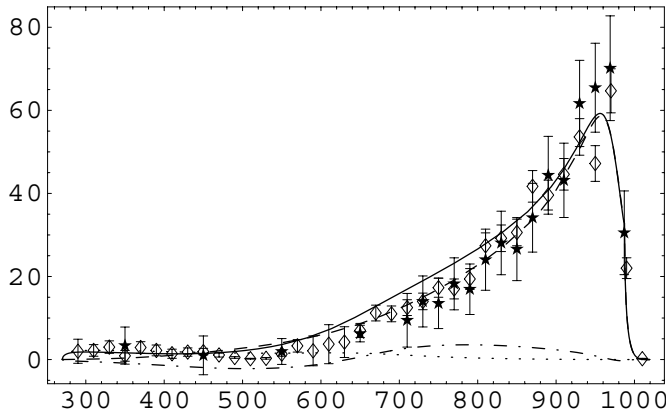
Fig. 1. $dB(\phi \to \pi^0\pi^0\gamma)/dm_{\pi^0\pi^0} \times 10^8$ (in units of MeV^{-1}) as a function of the dipion invariant mass $m_{\pi^0\pi^0}$ (in MeV). The dashed, dotted and dot-dashed lines correspond to the separate contributions from LσM, VMD and their interference, respectively. The solid line is the total result. We have taken $m_\sigma = 478$ MeV, $\Gamma_\sigma = 324$ MeV, $m_{f_0} = 980$ MeV, $\Gamma_{f_0} = 70$ MeV and $\phi_S = -6°$. Simple Breit–Wigner formulae have been used for the σ and f_0 propagators. Experimental data ($\star$) are taken from [5] and ($\diamond$) from [7]

contributes decisively to achieve this agreement. Indeed, for the integrated decay width one now obtains $\Gamma(\phi \to \pi^0\pi^0\gamma)_{\text{L}\sigma\text{M+VMD}} = 530$ eV and for the branching ratio

$$B(\phi \to \pi^0\pi^0\gamma)_{\text{L}\sigma\text{M+VMD}} = 1.19 \times 10^{-4}, \qquad (23)$$

quite in line with the experimental results quoted in (1) and (2). Without the VMD contribution, our result decreases only by some 10%, $B(\phi \to \pi^0\pi^0\gamma)_{\text{L}\sigma\text{M}} = 1.07 \times 10^{-4}$, but it still remains well above the value quoted in (8), $B(\phi \to \pi^0\pi^0\gamma)_\chi = 0.49 \times 10^{-4}$, which did not contain the crucial scalar contributions. In order to show the sensitivity of our treatment to the f_0 decay width and to the scalar mixing angle we have plotted in Fig. 2 our predictions for the $\pi^0\pi^0$ invariant mass distribution in $\phi \to \pi^0\pi^0\gamma$ for various values of Γ_{f_0} and ϕ_S. The prediction including chiral loops (without scalar poles) and VMD corrections is also shown for comparison. The invariant mass spectrum and the branching ratio are very sensitive to Γ_{f_0} and, even more, to ϕ_S. Present data on $\phi \to \pi^0\pi^0\gamma$ [5–7] favor $\phi_S \simeq -6°$ – which is in the expected range $-14° \leq \phi_S \leq -3°$ [26,27] – and $\Gamma_{f_0} \simeq 70$ MeV for the total width of the f_0 Breit–Wigner. This latter value is compatible with the PDG estimates, $\Gamma_{f_0} \simeq 40$–100 MeV, and somewhat above other recent values obtained with usual Breit–Wigner expressions: $\Gamma_{f_0} = 56 \pm 20$ MeV [6] or $44\pm2\pm2$ MeV [34]. Notice that in our approach $\phi_S \simeq -6°$ implies $\Gamma(f_0 \to \pi\pi) \simeq 35$ MeV accounting for some 50% of the total f_0 width. Other decay channels, such as $f_0 \to K\bar{K}$ for which our treatment predicts a large coupling, should account for the remaining 50%. But this requires a two-channel analysis around the $f_0 \to K\bar{K}$ threshold to which we now turn.

The original two-channel treatment of scalar resonances near the $K\bar{K}$ threshold due to Flatté [29] has been improved in [4,30] and is now widely applied in

experimental analyses of $V \to PP\gamma$ decays [5–7,9,35]. It amounts to the substitution of the simple expression $D_{f_0}(s) = s - m_{f_0}^2 + \text{i}m_{f_0}\Gamma_{f_0}$ by a complete one-loop f_0 propagator $D_{f_0}(s) = s - m_{f_0}^2 - \text{Re}\Pi(m_{f_0}^2) + \Pi(s)$, where the term $\Pi(s) - \text{Re}\Pi(m_{f_0}^2)$ takes into account the finite-width corrections due to the $\pi\pi$ and $K\bar{K}$ channels. Within our LσM approach, the $f_0 \to \pi\pi$ and $f_0 \to K\bar{K}$ partial widths can be expressed in terms of m_{f_0}, ϕ_S and accurately measured pseudoscalar masses and decay constants. For $m_{f_0} = 985$ MeV and $\phi_S = -9°$ we obtain the various curves shown in Fig. 3 which explain quite reasonably the available data. Integrating over the whole kinematical region one obtains $\Gamma(\phi \to \pi^0\pi^0\gamma) = 518$ eV and

$$B(\phi \to \pi^0\pi^0\gamma) = 1.16 \times 10^{-4}. \qquad (24)$$

The shape of the $\pi\pi$ mass spectrum depends strongly on the value for ϕ_S, as shown in Fig. 4, and follows the same pattern as previously shown in Fig. 2.

6 Comments and conclusions

The recent publication by experimental groups working at VEPP-2M [5,6] and DAΦNE [7] of the first data on the $\phi \to \pi^0\pi^0\gamma$ decay opens the possibility to improve our understanding of the controversial $I = 0$ scalar resonances below and around the 1 GeV mass region. In our present approach we deal with these scalar resonances by combining ChPT requirements with LσM dynamics. This treatment was first applied with some success to $\phi \to \pi^0\eta\gamma$ [19,20] and later to $\rho, \omega \to \pi^0\pi^0\gamma$ [21]. The main feature in our approach is the use of an amplitude which agrees with ChPT in the low part of the two-pseudoscalar invariant mass spectrum but which also includes the scalar poles dominating at higher invariant mass values. According to our final amplitude (12), the $\phi \to \pi^0\pi^0\gamma$ decay proceeds through a loop of charged kaons (as discussed, among others, in [3,4]) strongly coupled to the initial ϕ and to the final dipion system through scalar resonance formation in the s-channel. Additional vector meson contributions, acting as a kind of background to the previous and more interesting scalar exchange, have been recalculated in the framework of VMD thus confirming older results [10].

The measured values for the branching ratio $B(\phi \to \pi^0\pi^0\gamma)$ in (1) and (2) and the shape of the $\pi\pi$ mass spectrum [5–7] cannot be explained without a sizable contribution from the f_0, which depends on the expression of the f_0 propagator. If a simple Breit–Wigner expression is used, a good description of the data is achieved for $\Gamma_{f_0} \simeq 70$ MeV and $\phi_S \simeq -6°$. Although these two values are only marginally consistent within our model, the whole situation looks rather satisfactory if one takes into account the simplicity of our approach and the well-known difficulties one has to face in scalar meson physics. If a complete one-loop expression is used for the f_0 propagator [4,30], a fully consistent description of the data is achieved with $\phi_S \simeq -9°$. The range of values for ϕ_S is thus considerably restricted regardless of the expression adopted for the f_0 propagator.
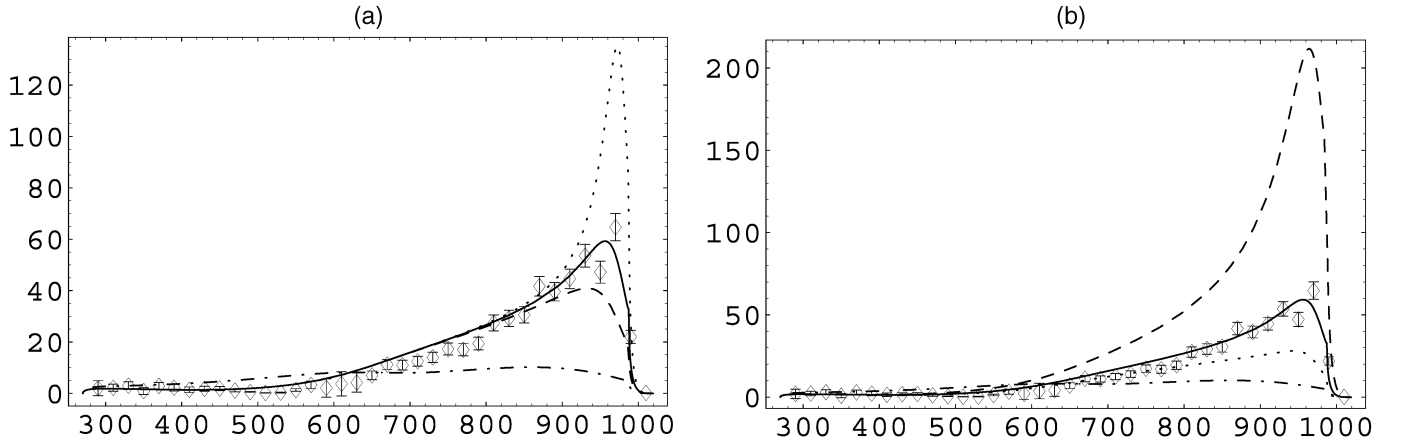
(a) (b)

Fig. 2a,b. $dB(\phi \to \pi^0\pi^0\gamma)/dm_{\pi^0\pi^0} \times 10^8$ (in units of MeV^{-1}) as a function of the dipion invariant mass $m_{\pi^0\pi^0}$ (in MeV). Simple Breit–Wigner formulae have been used for the σ and f_0 propagators. **a** Predictions for a fixed scalar mixing angle $\phi_S = -6°$ and $\Gamma_{f_0} = 40\,\mathrm{MeV}$ (*dotted line*), $\Gamma_{f_0} = 70\,\mathrm{MeV}$ (*solid line*) or $\Gamma_{f_0} = 100\,\mathrm{MeV}$ (*dashed line*). **b** Predictions for a fixed f_0 decay width $\Gamma_{f_0} = 70\,\mathrm{MeV}$ and $\phi_S = -3°$ (*dotted line*), $\phi_S = -6°$ (*solid line*) or $\phi_S = -14°$ (*dashed line*). The prediction including chiral loops and VMD corrections is also shown for comparison (*dot-dashed line*). Experimental data are taken from [7]
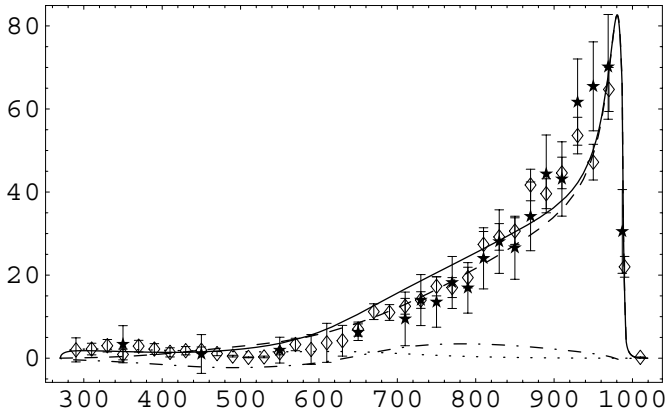

Fig. 3. $dB(\phi \to \pi^0\pi^0\gamma)/dm_{\pi^0\pi^0} \times 10^8$ (in units of MeV^{-1}) as a function of the dipion invariant mass $m_{\pi^0\pi^0}$ (in MeV). The dashed, dotted and dot-dashed lines correspond to the separate contributions from LσM, VMD and their interference, respectively. The solid line is the total result. For the σ propagator we use a simple Breit–Wigner expression with $m_\sigma = 478\,\mathrm{MeV}$ and $\Gamma_\sigma = 324\,\mathrm{MeV}$. For the f_0 propagator we use the complete one-loop expression [4,30] with $m_{f_0} = 985\,\mathrm{MeV}$ and $\phi_S = -9°$. Experimental data ($\star$) are taken from [5] and ($\diamond$) from [7]

Fig. 4. $dB(\phi \to \pi^0\pi^0\gamma)/dm_{\pi^0\pi^0} \times 10^8$ (in units of MeV^{-1}) as a function of the dipion invariant mass $m_{\pi^0\pi^0}$ (in MeV). The complete one-loop expression has been used for the f_0 propagator. Predictions for a fixed f_0 mass $m_{f_0} = 985\,\mathrm{MeV}$ and $\phi_S = -3°$ (*dotted line*), $\phi_S = -9°$ (*solid line*) or $\phi_S = -14°$ (*dashed line*). Experimental data are taken from [7]

In turn, such small values for $|\phi_S|$ imply a large $\sigma\pi\pi$ coupling which, in principle, should clearly become manifest in the low part of the dipion mass spectrum. But this is not the case in a LσM approach like ours where the $\sigma(500)$ contribution contains a $\sigma K\bar{K}$ coupling which is proportional to $m_\sigma^2 - m_K^2$ and thus almost vanishing. This depletion effect of the $\sigma(500)$ is somehow visible in the DAΦNE data which show very small values for $dB(\phi \to \pi^0\pi^0\gamma)/dm_{\pi\pi}$ in the $m_{\pi\pi} \simeq m_\sigma \simeq 500\,\mathrm{MeV}$ region. Indeed, the chiral-loop amplitude (without scalar poles) plus the VMD background are known to predict values [11] which are two or three times larger than the data in this range of masses. This feature, namely, the pre-
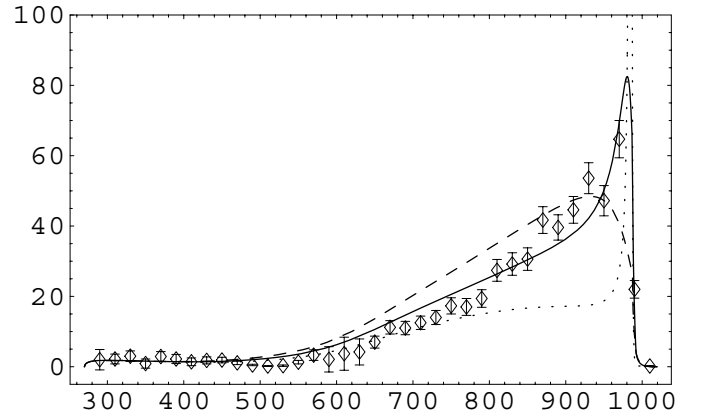
diction of small $\sigma K\bar{K}$ and large $\sigma\pi\pi$ couplings (because of their proportionality to $m_\sigma^2 - m_K^2$ and to $m_\sigma^2 - m_\pi^2$, respectively) is specific of our LσM approach. It admits a crucial and simple test. Contrary to what happens for $m_{\pi\pi}$ around 500 MeV in $\phi \to \pi^0\pi^0\gamma$ decays, data on $\rho \to \pi^0\pi^0\gamma$ – dominated in this case by pion rather than kaon loops – should show a non-suppressed $\sigma(500)$ contribution around $m_{\pi\pi} \simeq 500\,\mathrm{MeV}$ [36].

In summary, the recently published data on $\phi \to \pi^0\pi^0\gamma$ decays can be reasonably described in a rather simple context based on the complementary between ChPT and the LσM. Future refinements of our approach and comparison with forthcoming and more accurate data should contribute considerably to the clarification of one of the most challenging aspects of present day hadron physics, namely, the structure of the lowest lying scalar states.

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